Graph Theory and Applications Beycan Kahraman
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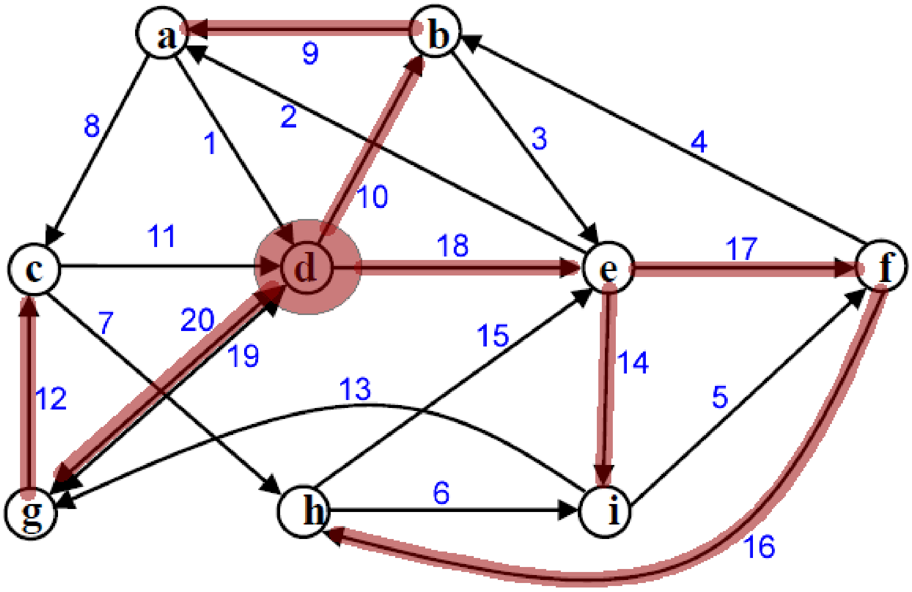
Homework 3 **05.12.2008**

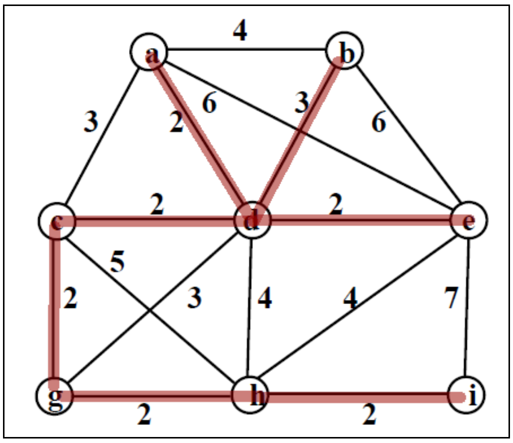
1. Find an Euler tour of the digraph below.



- Each vertex has even degree; therefore, we could find an Euler tour with Fleury's algorithm.
- First of all, spanning out-tree of the digraph could be created as shown in the graph with root **d**.
- We will create the tour in reverse direction, by preferring the edges that are not in the spanning tree at first.
- Therefore, the algorithm could find one of the possible solution as:
  d,1,a,2,e,3,b,4,f,5,i,6,h,7,c,8,a,9,b,10,d,11,c,12,g,13,i,14,e,15,h,16,f,17,e,18,d,19,g,20,d
- In the end, the Euler tour is the reverse order of the series:
  d,20,**g,**19,**d,**18,**e,**17,**f,**16,**h,**15,**e,**14,**i,**13,**g,**12,**c,**11,**d,**10,**b,**9,**a,**8,**c,**7,**h,**6,**i,**5,**f,**4,**b,**3,**e,**2,**a,**1,**d**

2. Solve the second version of TSP for the graph below. What is the cycle length?
(You can use twice around the minimum spanning tree algorithm.) Bonus (+10): Find the exact solution.

Twice around the minimum spanning tree algorithm could work only on complete graphs. Therefore, we should decide some of the priorities so that we could use the proposed algorithm.



In addition to this, the given graph does not satisfy the triangular equality as in (b, d, e) triangle. Therefore, the algorithm could not guarantee finding twice around the MST.

Choosing **d** as a root for MST, using DFS, one of the possible solution is: **d,**3,**b,**4,**a,**3,**c,**2,**g,**2,**h,**2,**i,**7,**e,**2,**d** **[25]**

Let |ie|=100, the algorithm could not guarantee 2 x MST.
Another examples: |ac| = 200, |ab| = 50, …
Triangle equality makes them smaller than 2 **x** MST edges.

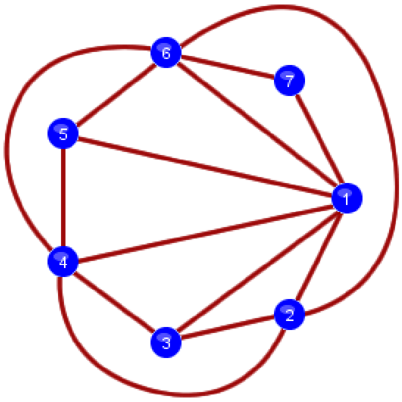
Another one (root **a**): **a,**2,**d,**2,**c,**2,**g,**2,**h,**2,**i,**7,**e,**6,**b,**4,**a** **[27]**

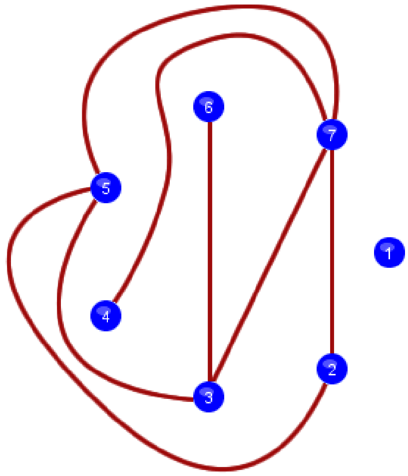
Since the graph is not complete: a,2,d,3,b,6,e,?,c the algorithm will fail in choosing the given order.
It is possible for twice around MST algorithm to choose such an order.

The exact solution could be achieved with brute forcing as: **a,** 4, **b,** 3, **d,** 2, **e,** 7, **i,** 2, **h,** 2, **g,** 2, **c,** 3, **a** **[25]**

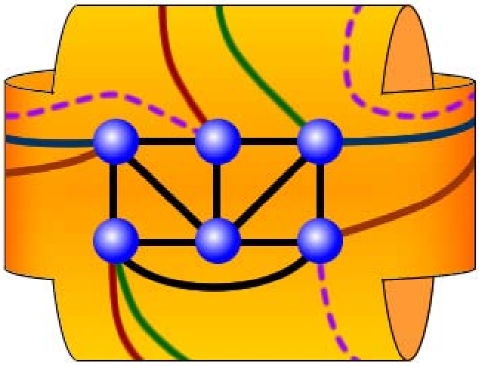
3. What is the thickness of K_7? Draw the planar sub-graphs of K_7.

K_7 has 7 vertices and $\frac{6x7}{2} = 21$ edges. Therefore, $T \geq \left\lceil \frac{e}{3n-6} \right\rceil = \left\lceil \frac{21}{15} \right\rceil = 2$

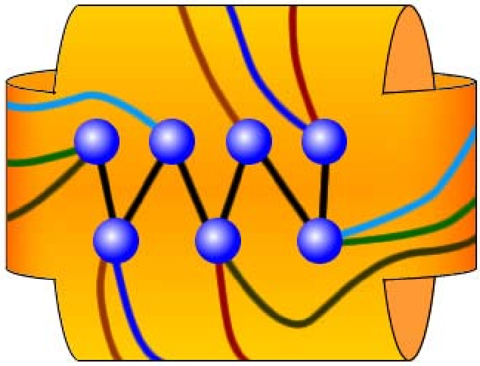




4. Embed K_6 and $K_{4,3}$ on a torus.
(You can represent the surface of a torus, as a rectangle. Imagine that the left and right sides can be folded back and glued together. Also the upper and lower sides of the rectangle can be folded back and glued together.)



K_6 on a torus



$K_{4,3}$ on a torus